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DISTORTION LOAD MODELLING FOR DISTRIBUTION SYSTEM HARMONIC STUDIES

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Abstract

A study is made of the current harmonics drawn by capacitor-filtered rectifier loads. It is shown that dc side ripple and supply resistance have only minor effects on the current spectra. Graphical results obtained by simulation are given for the line current spectra of a single phase rectifier for different supply impedances and approximated by curve-fitting with a single equation. The same equation is shown to apply to three phase rectifiers with triplen harmonics taken to be zero. Some results are obtained for combining many small rectifiers in parallel when they all have the same ripple voltage. The concept of partial-self compensation is examined and explained in the light of the results obtained for supply current spectra.

1. INTRODUCTION

In the past, harmonic problems generally arose in installations where there was one dominant distorting load, usually a form of controlled rectifier and an inductor-filtered load such as a dc motor. This has a current harmonic spectrum closely following a 1/n law. Because of this, many harmonic studies (and analysis programs), assume a similar variation for the general distorting load. The assumption has been incorporated into the Australian harmonic standards AS 2279.2 where Figure 1, showing the maximum converter size which can be connected for a given short circuit level, has been calculated using such a model [1].

Recently attention has been drawn to a different form of distorting load such as occurs in commercial office buildings and consisting of many small rectifiers of the capacitor-filtered type [2, 3]. These may be single phase rectifiers (office equipment, high efficiency lighting) or three phase (ac variable speed drives). This type of load has a different type of waveform as shown by the PSPICE computed waveforms in Fig. 1 for a single phase rectifier case. The modelling of the capacitor-filtered load is also complicated by its presence as many small units, all sharing a common source impedance.

Surprisingly few papers have treated this rectifier in detail, and none give the results in a form convenient for estimating the current harmonics which a particular installation will draw. Sakui [4] describes a three phase rectifier with a small inductance on the dc side and obtains approximate analytical results by assuming the dc current to be continuous. Kelley [5] examines a single phase rectifier without source impedance and determines the optimum value of dc side inductance for minimum harmonic current. A three phase rectifier with source inductance and small or zero dc side inductance is discussed in Sanders [6]. The results for V_5 and V_{THD} at the point of common coupling are shown in several graphs for different short circuit ratios (ratio of supply fault level to rectifier load).

Mansoor [7-9] look at the effect of combining many single phase rectifiers with a common source impedance and diversity due to slight differences in phase angle. The approach is partially analytical with conduction angles having to be calculated by numerical methods, thus the results are not useful for general application. Of particular interest is a finding in [8] called partial self-compensation. The peaky waveform of the rectifier current has the effect of flattening the supply voltage waveform. This extends the rectifier conduction period reducing its harmonic content. It is claimed that as rectifiers are connected in sequence to an installation, their relative harmonic contribution

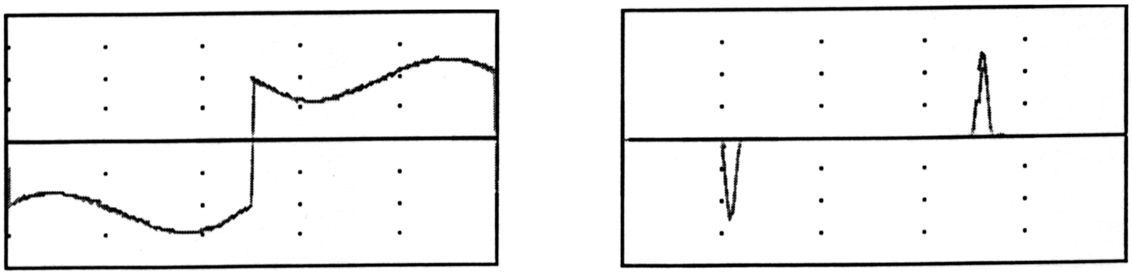

(a) (b)

Fig. 1 Current waveforms for (a) inductor-filtered and (b) capacitor-filtered rectifiers decreases. This seems to suggest that a large number of small rectifiers give a lower harmonic current than a single rectifier of the same rating.

The aim of this paper is to develop accurate models of capacitor-filtered rectifiers for harmonic studies. The study will concentrate on the harmonic current spectrum. Effects to be considered are capacitor size, Short Circuit Ratio (SCR), X-to-R ratio of source impedance, number of phases and the interaction of several units. The effect of a small dc-side inductance, although used in some loads, is not considered.

2. METHODOLOGY

It is worth reviewing the behaviour of the traditional inductor-filtered rectifier as a basis for later discussions of the capacitor-filtered rectifier. Harmonic current I_n, is proportional to 1/n, where n is the harmonic order, for both the single phase and three phase circuits. The supply impedance has no first order effect on this expression for the dominant low order harmonics. The harmonic spectrum of three phase inductor-filtered rectifiers are the same as the single phase rectifiers without the triplen harmonics (3, 6, 9, ...). Several inductor-filtered rectifiers at the same point can be combined to give a single equivalence equal to the sum of the separate ratings because of the current-stiff nature of the DC side.

Except in AC variable speed drives, capacitor-filtered rectifiers occur in large numbers of small ratings. The harmonic model for the capacitor-filtered rectifier does not follow the inductor-filtered rectifier model of 1/n but falls off more rapidly at higher frequencies. The low frequency harmonics can be very large and possibly larger than the fundamental component. There is no obvious theoretical reason why several small rectifiers can be represented by a single rectifier of the combined rating because of the voltage-stiff nature of the DC side.

The effect of the filter capacitor size on the harmonics produced in the circuit has been carried out at different levels of DC ripple voltage. The effect of the system line impedance ratio was studied by varying X-to-R across a range to represent LV and HV supplies. The Short Circuit Ratio was varied to more than cover the range of values commonly encountered. Single and three phase rectifiers with the same SCR were compared. Partial self-compensation was investigated by comparing a single capacitor-filtered rectifier with several rectifiers of the same rating.

Single and three phase studies have been made on 240 V and 415 V voltages respectively. In all cases the rectifier dc load was assumed to be 100 W. It is assumed that the results can be applied to any other combination of voltage levels and power ratings having the same per unit values.

Due to difficulty in mathematically solving the capacitor-filtered rectifier equations, simulation techniques using PSPICE were used. The dc load was simulated in the form of $I_{dc} = A\text{-}BV_{do}$ to approximate a constant power load as would be expected from switched mode power supplies and variable speed drives over short periods. In the case of a single phase rectifier, the expression $0.64\text{-}0.001V_{dc}$ was found to be sufficiently accurate over the range of voltages which occurred.

Care is needed in using PSPICE for this type of problem. The default diode parameters have to be modified (IS = 1E-4, RS = 0.01) and the tolerance options reset. Most results were obtained with RELTOL = 0.01, VNTOL = 0.5, ABSTOL = 0.01. The line current harmonic spectrum was calculated using Probe and exported to EXCEL for further processing.

Curve-fitting techniques were then used to determine an expression for the line current harmonics including the effects of all important parameters. It was not possible to obtain expressions which were accurate for the full range of SCRs considered. Where compromise was required, accuracy was emphasised more at low values of SCR where it is more likely that harmonic voltage limits might be exceeded.

3. EFFECT OF CAPACITOR SIZE AND SUPPLY RESISTANCE

The parameters which can affect line current harmonics are rectifier capacitance , supply X-to-R ratio and the short-circuit ratio (SCR) defined as

$$\mathrm{SCR} = \frac{\text{Supplyfaultlevel}}{\text{Rectifierloadrating}} \qquad (1)$$

Sanders [6] suggests that the later two have minor importance. It was decided to check this by simulation of some cases to cover the expected range of parameter variations. These were assumed to be as follows

1. SCR was taken to vary from 10^1 to 10^7. This results in line reactances of 183 mH to 183 nH for a 100 W single phase unit.

	Voltage Ripple %		
SCR	0.1%	1.0%	10%
10^1	6.5 mF	670μF	65μF
10^4	8.0 mF	800μF	80μF
10^7	8.0 mF	800μF	80μF

Table I DC Filter Capacitor Size for Different SCRs and Voltage Ripple

	Voltage Ripple %					
	0.1%		1.0%		10%	
SCR	V_{THD} %	I_{THD} %	V_{THD} %	I_{THD} %	V_{THD} %	I_{THD} %
10^1	23	52	23	52	24	55
10^4	0.28	230	0.29	240	0.42	230
10^7	0.20	400	0.06	330	0.0016	180

Table II Current and Voltage Total Harmonic Distortion

2. The filter capacitance variations were assumed to cover a range giving DC ripple voltages of 0.1% to 10%.
3. X-to-R was assumed to vary from 25 (high voltage system) to 1:1 (distribution system)

An initial study looked at the dependence of C on the SCR. Supply resistance was held at zero and C varied to give ripple voltages of 0.1%, 1% and 10%. SCR was also varied with values 10^1, 10^4 and 10^7 giving nine cases in total shown in Table I. It can be seen that the capacitor size for a given ripple voltage is not very sensitive to the SCR. For example, a change in SCR of 10^7:1 requires the capacitor to be varied by no more than 1.2:1. The size of C varies inversely as the ripple as expected.

In a second study, the THD in the supply voltage and current were determined for the nine combinations of SCR and capacitor (or ripple voltage). Simulation results for the circuit are tabulated in Table II showing the very strong dependence on SCR and the weak dependence on C.

Based on the total harmonic distortion, capacitor voltage does not have a significant effect on circuit performance. All subsequent studies have been performed with the filter capacitor chosen to give a 1% DC voltage ripple. The X-to-R supply impedance ratio was examined at 1:1, 5:1 and 25:1. Supply SCR was varied from 10^1 to 10^7 as before and resulted in harmonic current and voltage distortion shown in Table III. The filter capacitor was varied in accordance with Table I to keep the DC ripple at 1% for each SCR.

It can be seen from Table III that both SCR and supply X-to-R have effects on the THD. Considering the lower two values of SCR where accuracy is most required, X-to-R can vary the voltage THD by 1:1.5 while SCR can vary it by almost 100:1. To keep the problem tractable, it seemed warranted to ignore the

	Supply Impedance X to R Ratio					
	1:1		5:1		25:1	
SCR	V_{THD} %	I_{THD} %	V_{THD} %	I_{THD} %	V_{THD} %	I_{THD} %
10^1	15	63	20	55	22	53
10^4	0.19	260	0.26	240	0.29	240
10^7	0.0015	330	0.0015	320	0.0023	320

Table III Total Harmonic Distortion for Varying SCR

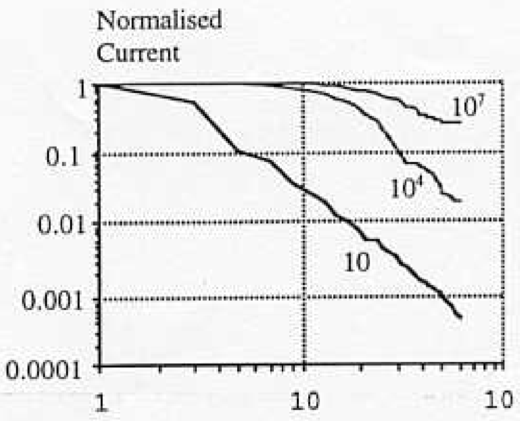


Fig. 2 Harmonic Current Spectra for Varying SCR

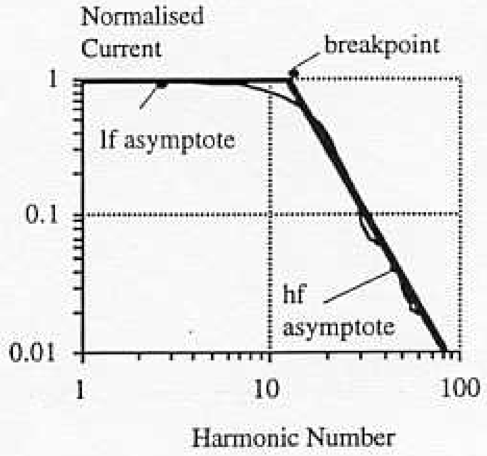


Fig. 3 Asymptotic approximation to Bode plot for SCR = 10^4

effects of X-to-R variations, and a value of 25 was used in all subsequent simulations.

4. EFFECT OF SCR ON SINGLE PHASE RECTIFIER OPERATION

With the nominal values of C and X-to-R given in the previous section, harmonic current spectra were calculated for SCRs from 10^1 to 10^7 in steps of one decade. Some results are given in Fig. 2 plotted using log-log axes. A line graph is given rather than a bar graph to allow the trends to be more obvious. This has the minor disadvantages that it incorrectly suggests line current values at even harmonics and gives unevenness in the graph at low harmonics where widely spaced points are joined.

Fig. 2 shows the following features:

1. The harmonics fall off in a more complicated way than a 1/n law. They fall off more slowly for low n and then more rapidly at high n
2. As expected, harmonic currents reduce with reduced SCR.
3. There is a significant rate of change of the harmonic spectrum with SCR when the SCR is about 10, but only a small rate of change for SCRs of 10^4 and greater.

An attempt has been made to summarise these results by means of a simplified equation to replace the 1/n assumption. As shown in Fig. 3 for the case of SCR = 10^4, the spectra can be approximated by asymptotes with slopes of zero for low frequency and with breakpoints and high frequency slopes varying with SCR. The asymptotes for the high values of SCR were obtained by drawing a mean straight line through the curve concentrating on obtaining a good fit particularly at the higher values of current. Values of breakpoints and slopes were found as given in Table IV. The slopes correspond to a power law, for example a slope of 2.5 corresponds to a variation of $n^{-2.5}$.

SCR	Breakpoint	Slope
10	3	2.5
100	5	2.5
1000	9	2.7
10000	17	3.0
10000000	30	2.5

Table IV Breakpoints and slopes vs SCR

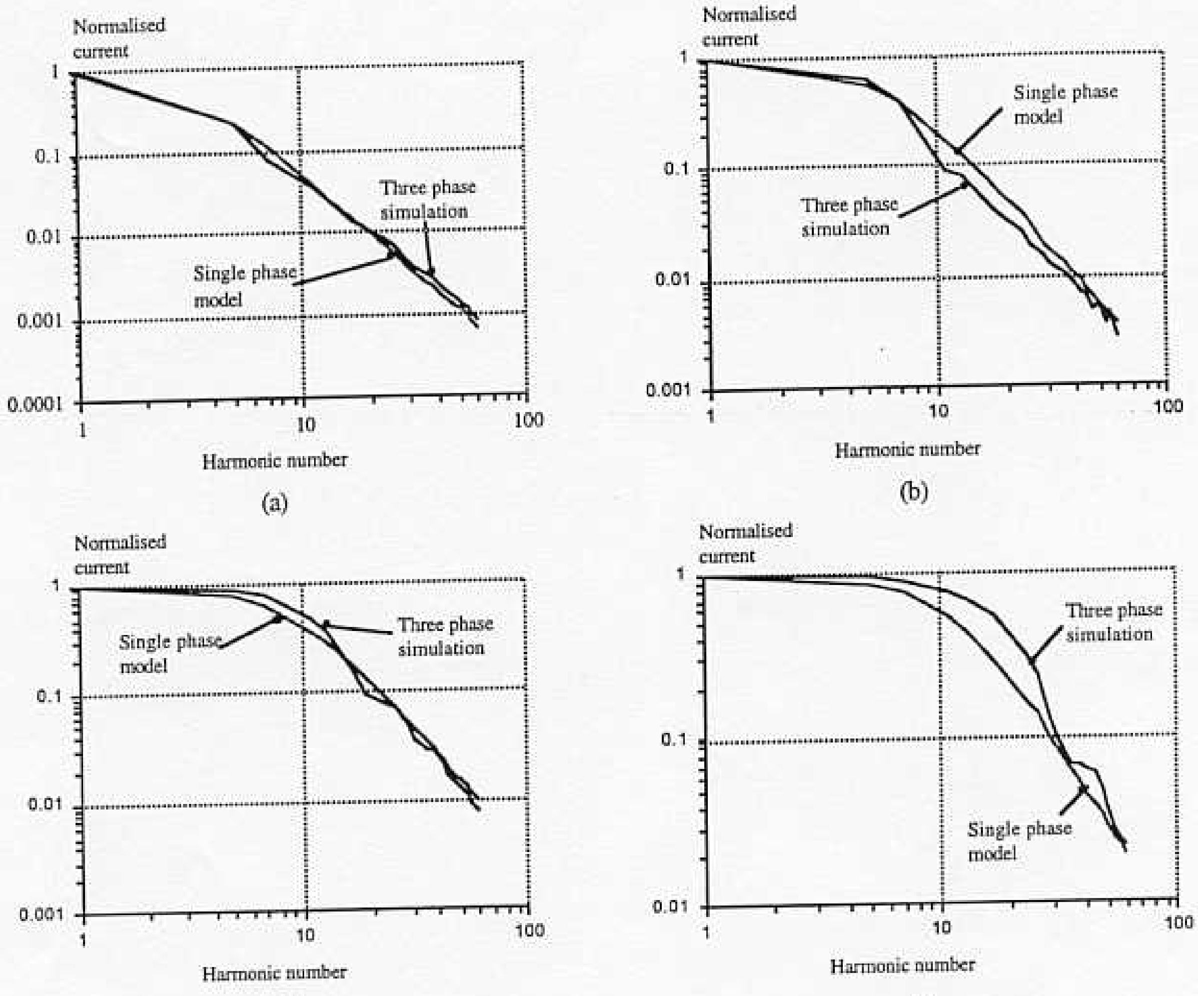
Notice that the slope varies by only a small amount with a slight reduction at very high values of SCR. The high frequency fall off is considerably steeper than for inductor-filtered ratifiers The breakpoints vary to a first approximation with the logarithm of the SCR. A search was then made for a function of the following form

$$I_n(SCR) = \frac{1}{1+\left(\frac{n}{a\log SCR}\right)^b} \tag{2}$$

where a and b are constants. A least squares optimisation was made using the Solver in Excel giving a = 3 and b = 2.7, hence

$$I_n(SCR) = \frac{1}{1+\left(\frac{n}{3\log SCR}\right)^{2.7}} \tag{3}$$

This function has been found to agree with the simulated results with an accuracy of better than 10% at SCRs of 10^4 or lower where accuracy is most important. Agreement near the breakpoint is much better than is suggested by the asymptotes in Fig. 3.



(a) (b) (c) (d)

Fig. 4 Line current spectra for three phase rectifier compared with analytical model
(a) SCR = 10, (b) SCR = 10^2, (c) SCR = 10^3, (d) SCR = 10^4

5. THREE PHASE CAPACITOR-FILTERED RECTIFIER

It is assumed that the three phase rectifier is similarly insensitive to capacitor size and X-to-R ratio. The circuit has been set up with a similar rating of 100 W supplied at 415 V line-line and simulated for a similar range of SCRs. Graphs of the normalised harmonic current distortion for both single and three phase circuits are shown in Fig. 4.

As previously explained, the graphs are drawn as smooth lines through the non-zero harmonics and should not be interpreted as suggesting that the three phase rectifier draws triplen or even harmonics. The analytical expression of Eqn(3) have been superimposed on the graphs and show a good agreement except at high values of SCR (10^4 and above).

6. PARTIAL SELF-COMPENSATION

The effect of a reduction in harmonic current distortion for n rectifiers of 1/n rating of the full size rectifier, has been described as partial self-compensation [8]. This effect was examined using a single phase model with (i) a 100W load, (ii) five rectifiers of 20W rating. An SCR of 10^3 was used. Simulation results for the two cases were the same providing the capacitor size was reduced to one fifth in Case (ii) to keep the voltage ripple the same.

The situation becomes rather confusing if each rectifier has a different value of voltage ripple, since the first rectifier turning on reduces the voltage level and affects the time at which subsequent rectifiers will begin conduction. It was found that the line current was within about 5% of Cases (i), (ii) above in the small number of combinations which were examined. There is scope for a more detailed study where rectifier sizes and voltage ripple might differ between units. With this proviso, it appears that several rectifier units can be replaced by a single unit of the combined rating when using Eqn(3) to calculate installation current harmonics.

This conclusion does not appear at first sight to be consistent with Mansoor's comments on partial self-compensation [8]. This can be explained by following through Mansoor's work in more detail. He considered one rectifier connected to a system and the effect on current THD by connecting additional units. Mansoor's approach has the problem that the SCR is being reduced with the connection of each unit, introducing other effects which mask possible interactions between rectifier units.

For example, a 100 W rectifier connected to a point where the fault level is 100 kVA has an SCR of 10^3 and Eqn(3) gives a 5th harmonic current of 79%. Adding a second rectifier reduces the SCR to 500 and the normalised current reduces to 75%. Further calculation for five rectifiers gives a reduction to 67%,

Mansoor shows the current THD reducing by a factor of 2 as the number of units is increased from 1 to 15. This can be explained by assuming an initial SCR of 300 without any recourse to partial self-compensation as a special feature in rectifier interaction.

7. CONCLUSIONS

Simulation results have shown that the capacitor-filtered rectifier has a line current spectra which is insensitive to capacitor size and supply resistance. However, it is sensitive to source impedance and has a shape which is more complicated than the simple 1/n rule commonly used with inductor-filtered rectifiers. A systematic study has been made of the dependence over a range of useful values and the results summarised to adequate accuracy in one equation. The same equation has been shown to apply to three phase rectifiers if the triplen harmonics are treated separately and taken as zero.

A number of rectifiers having identical voltage ripple can be replaced by a single rectifier for harmonic calculation purposes. The concept of partial self-compensation is not needed to explain the observation that rectifier THD reduces as the number of rectifiers increases.

8. ACKNOWLEDGMENTS

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9. REFERENCES

[1] AS 2279.2-1991 "Disturbances in mains supply networks. Pt 2: Limitations of harmonics caused by industrial equipment", Standards Australia, 1991

[2] J. Lai and T. Key, "Effectiveness of harmonic mitigation equipment for commercial office buildings", IEEE Trans. Industry Applications, Vol. 33, No. 4, pp. 1104-1110

[3] K. Johnson and R. Zavadil, "Assessing the impacts of non-linear loads on power quality in commercial buildings - An overview", Proc. Industry Applications Society Annual Meeting, 1991, pp. 1863-1869

[4] M. Sakui, H. Fujita and M. Shiyoya, "A method of calculating harmonic currents of a three phase bridge uncontrolled rectifier with DC filter", IEEE Trans. Industrial Electronics, Vol. 36, No. 3, Aug. 1989, pp. 434-440

[5] A.W. Kelley and W.F. Yadusky, "Rectifier design for minimum line current harmonics and maximum power factor", IEEE Trans. Power Electronics, Vol. 7, No. 2, Apr. 1992, pp. 332-341

[6] G.J. Sanders, "Supply harmonics caused by AC variable frequency drives", Pacific Power Centre for Energy Efficiency, University of Wollongong, Motor and drive technical note No. 1, 1992

[7] A. Mansoor, W.M. Grady, A.H. Chowdhury and M.J. Samotyj, "An investigation of harmonics attenuation and diversity among distributed single phase power electronic loads", IEEE Trans. Power Delivery, Vol. 10, No. 1, Jan. 1995, pp. 467-473

[8] A. Mansoor, W.M. Grady, R.S. Tahallam, M.T. Doyle, S.D. Krein and M.J. Samotyj, "Effect of supply voltage harmonics on the input current of single phase diode bridge rectifier loads", IEEE Trans. Power Delivery, Vol. 10, No. 3, Jul. 1995, pp. 1416-1422

[9] A. Mansoor, W.. Grady, P.T. Staats, R.S. Thallam, M.T. Doyle and M.J. Samotyj, "Predicting the net harmonic currents produced by large numbers of distributed single phase computer loads", IEEE Trans. Power Delivery, Vol. 10, No. 4, Oct. 1995, pp. 2001-2006